State, County and Minor Civil Division Population Projection Methodology: A Brief Overview

Background

Wisconsin's Department of Administration has developed population projections based on the U.S. Census Bureau's 2020 counts. Projections results are summarized in a separate document.[1] Projections results spreadsheets are available on the web page that hosts this document.[2]

County Population Projections: The Modified Hamilton-Perry Method

An abbreviated description of the Hamilton-Perry method appears here. Detailed discussion can be found in a seminal paper forecasting Clark County, Nevada census tract population.[3] Another often-cited paper applies the Hamilton-Perry method to Massachusetts small-area population estimates.[4]

For most age groups, the Hamilton-Perry method uses a ratio (called the Cohort Change Ratio) from the two most recent U.S. Census Bureau counts to project future population. For a specific county, the formula for calculating the female population between 30 and 34 in 2030 looks like this:[5]

$$Females\ aged\ 30\text{-}34\ in\ 2030 = \frac{Females\ aged\ 30\text{-}34\ in\ 2020}{Females\ aged\ 20\text{-}24\ in\ 2010} \times\ Females\ aged\ 20\text{-}24\ in\ 2020$$

Intuitively, it's not hard to see that the females aged 30-34 years in 2030 would have been aged 20-24 years in 2020. The fraction in the formula above is called the Cohort Change Ratio. Cohort Change Ratios were calculated for each county in each applicable age group. A cohort is simply a group of people who were 20-24 in 2010 and 30-34 in 2020. The Cohort Change Ratio calculates how much this group's numbers increased or decreased over the prior decade (2010-2020). The Hamilton-Perry

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1 See State and County Population Projections 2020-2050: A Brief Summary of Findings, https://doa.wi.gov/DIR/Summary_Pop_Proj_2020_2050.pdf.

2 See state and county age-sex population projections, https://doa.wi.gov/DIR/Proj_Co_State_2020_2050.xlsx See also, Minor Civil Division and Municipal Population Projections 2020-2050, https://doa.wi.gov/DIR/Proj_Muni_2020_2050.xlsx.

3 Swanson, D. A., Schlottmann, A., & Schmidt, B. (2010). "Forecasting the population of census tracts by age and sex: An example of the Hamilton–Perry method in action." *Population Research and Policy Review* 29(1), 47–63. See generally https://link.springer.com/article/10.1007/s11113-009-9144-7. Specifically, see https://link.springer.com/content/pdf/10.1007/s11113-009-9144-7.pdf.

4 See Strate, S., Renski, H., Peake, T., Murphy, J.J., & Zaldonis, P. (2016). "Small Area Population Estimates for 2011 through 2020." *UMass Donahue Institute.* https://donahue.umass.edu/documents/UMDI_Small_Area_Population_Estimates_for_2011_through_2020_2016.09.23%281%29_.pdf.

5 Traditionally, demographic methods assumed that longer lifespans for people categorized "females" and no births to people categorized as "males" justified the use of these binary categories. Population projections rely on population counts and fertility rates that follow this convention. Population projections to follow suit without endorsing the convention.

method's Cohort Change Ratio combines the effects of migration and mortality into a single fraction and uses that fraction to project future population change. Mortality can only affect population change negatively; migration can affect population change positively or negatively.

The people aged 85-and-over in 2020 include people who, in 2010, were aged 75-79, or 80-84 or 85-and-over. (Similarly, the people aged 85-and-over in 2030 will include people from these three groups in 2020.) For these reasons, the 85-and-over projection formula looks slightly different:

$$Females\ aged\ 85+\ in\ 2030 = \frac{Females\ aged\ 85+\ in\ 2020}{Females\ aged\ 75+\ in\ 2010} \times\ Females\ aged\ 75+\ in\ 2020$$

The Cohort Change Ratio is driven by migration and mortality. Both change factors vary by age and vary by sex. (Females have longer life expectancy and lower mortality early in the oldest age groups; people aged 20-24 don't have migration patterns identical to people aged 80-84.) For these reasons, individual Cohort Change Ratios were calculated for females and for males, for each of Wisconsin's 72 counties, for 16 age groups (10-14, 15-19, … 80-84, and 85-and-over). People who, in 2030, will be aged 0-4 or 5-9, had not been born yet in 2020, so their numbers were projected with fertility projections discussed below (pages 3-5).

Excess Deaths

Wisconsin's total deaths were above trend in calendar years 2020 and 2021 (arguably in 2022 as well). The Department of Administration calculated excess death rates for each age group 0-4 through 70-74. To calculate excess deaths, the reference data came from Centers for Disease Control and Prevention's web page for National Center for Health Statistics, specifically the life table reports.[6] That life table data is for single-year-of-age cohorts and single-year intervals (e.g. 2019[7] and 2020[8]), so the complete life tables had to be abridged to 5-year age groups to be compatible with the population projections. The differences between the 2019 life table and the 2020 life table were presumed to be "excess death" rates, largely attributable to covid-era health issues. These excess death rates were used to develop a modified 2020 base used to calculate 2030 population projections. This modified base does not appear in the published files and does not affect 2040 projections or 2050 projections. While many of the deaths in the younger age groups would have been less likely in the absence of covid-era health issues, the 80-84 group and the 85-and-over group may not have seen such a dramatic change in its rate of survival to 2030, so those two groups were excluded from excess death calculations.

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[6] See generally https://www.cdc.gov/nchs/products/life_tables.htm.

[7] See https://ftp.cdc.gov/pub/Health_Statistics/NCHS/Publications/NVSR/70-18/.

[8] See https://ftp.cdc.gov/pub/Health_Statistics/NCHS/Publications/NVSR/71-02/.

Fertility: Child-Woman Ratio's Shortcomings

If the Department of Administration had not modified the method, the Hamilton-Perry Method would have projected children born over the course of a decade by using a Child-Woman Ratio.

Two issues arise. If fertility rates change over the projection period, the Child-Woman Ratio will not capture that change. A thought experiment illustrates the second issue. Suppose the number of females aged 15-44 in 2020 and remains similar in 2030, but this age group's age distribution shifts over time. Suppose the females aged 15-44 in 2020 are mainly in higher-fertility age groups in 2020 (say, ages 25-29 and ages 30-34). Further suppose that females aged 15-44 in 2030 shift to lower-fertility age groups in 2030 (say, ages 35-39 and ages 40-44). The Child-Woman Ratio would not address any shift from higher-fertility age groups to lower-fertility age groups.

Fertility: Age-Specific Fertility Rates Trended

Instead of the Child-Woman Ratio's broad characterization of fertility, the population projections used each county's Age-Specific Fertility Rates.

$$Age\text{-}Specific\ Fertility\ Rate = \frac{Number\ of\ births\ to\ females\ in\ the\ age\ group}{Number\ of\ females\ in\ the\ age\ group} \times 1{,}000$$

Another way to think of the Age-Specific Fertility Rate is the number of children born to women in an age group per thousand women in that age group. The Age-Specific Fertility Rate graph below includes a data point of 86.9 for the years 1991-1995, for Brown County females aged 30-34. This means that, for every 1,000 Brown County females between 30 and 34 years old, roughly 86.9 births occurred per year between 1991 and 1995.

In January 2024, Age-Specific Fertility Rate data was gathered.[9] To reduce data volatility associated with smaller age groups or shorter time periods, the Age-Specific Fertility Rates gathered for the population projections used five-year age groups (15-19, 20-24 … 35-39, and 40-44) and used five-year time periods (1991-1995, 1996-2000, … 2016-2020). To minimize distortions, births to people under 15 were included in the Age-Specific Fertility Rate for ages 15-19 and births to people 45-and-over were included in the Age-Specific Fertility Rates for ages 40-44; the Wisconsin Department of Health Services provided this data. Over time, the Wisconsin Department of Health Services may receive more information about births that occurred in Wisconsin counties and may receive more information about estimated number of females in each age group, so neither the Age-Specific Fertility Rate's numerator nor its denominator are etched in stone.

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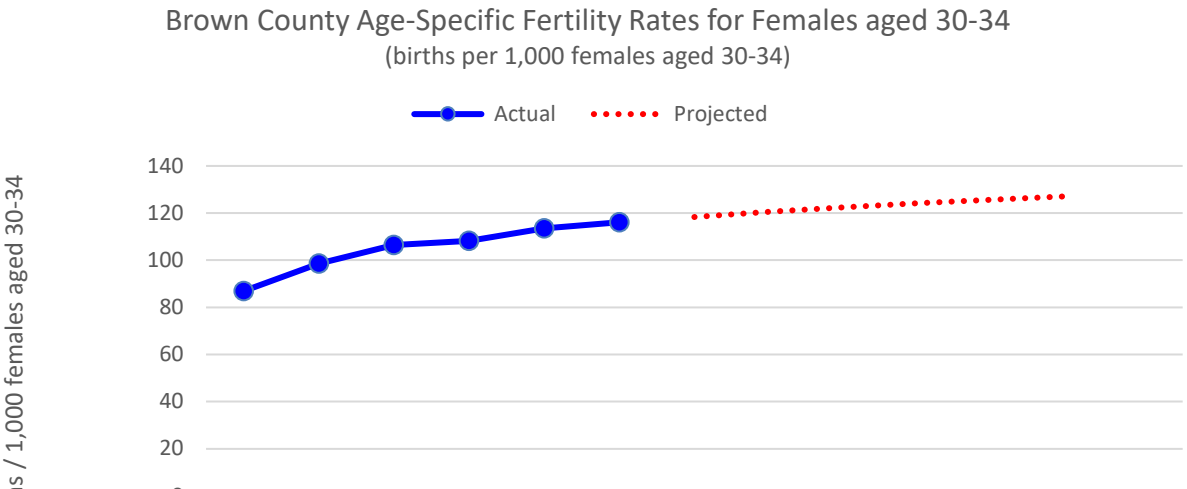
[9] Some data came from the Wisconsin Interactive Statistics on Health fertility form https://www.dhs.wisconsin.gov/wish/fertility/form.htm. Some data was specially requested from Wisconsin's Department of Health Services, Vital Records Data Management and Support Unit.

For each county, projections needed Age-Specific Fertility Rates for six age groups (ages 15-19, 20-24, 25-29 30-34, 35-39, and 40-44), for each of the six 5-year segments of the projections period (4/1/2020-3/31/2025; 4/1/2025-3/31/2030; 4/1/2030-3/31/2035; 4/1/2035-3/31/2040; 4/1/2040-3/31/2045; and 4/1/2045-3/31/2050). The graph below illustrates Brown County Age-Specific-Fertility Rates for females aged 30-34.



	1991-1995	1996-2000	2001-2005	2006-2010	2011-2015	2016-2020	2020-2025	2025-2030	2030-2035	2035-2040	2040-2045	2045-2050
Actual	86.9	98.6	106.5	108.2	113.5	116.1						
Projected							118.3	120.5	122.5	124.2	125.7	127.2

For the task of projecting rates forward, four methods were considered (natural logarithm, linear, power, and exponential). All four methods' parameters were calculated for all six age groups in all 72 counties. Age-Specific Fertility Rates were projected using the trend that best fit the historical data (highest R-squared measure), applying certain constraints to avoid impossible or very implausible outcomes. Unrounded, exact trend parameters were calculated.

Fertility: Applying Constraints

In some cases, a group (particularly the 15-19 age group) experienced Age-Specific Fertility Rate decreases in several counties. So much so that the trend with the best fit (highest R-squared measure) would sometimes suggest Age-Specific Fertility Rates below zero. There cannot be fewer than zero births per 1,000 females in an age group, so below-zero Age-Specific Fertility Rates are impossible. In these cases, a substitution was made. The last above-zero Age-Specific Fertility Rate would simply be halved in the next period, then that rate would be halved again in the next period, if necessary. This stays true to the general idea of a significant decrease without dipping into impossible (below-zero) territory.

It seemed reasonable to assume that a county could have high Age-Specific Fertility Rates in a particular age group and still experience fertility increases in that age group (up to a point). It seemed less reasonable to allow unlimited fertility rate increases between 2020 and 2050. For ages 20-24, Age-Specific Fertility Rates were allowed to rise, but not past 125 births per 1,000 females aged 20-24. For ages 25-29 and ages 30-34, Age-Specific Fertility Rates were allowed to rise, but not past 200 births per 1,000 females aged 25-29 (or aged 30-34). For ages 35-39, and Age-Specific Fertility Rates were allowed to rise, but not past 100 births per 1,000 females aged 35-39. These caps were set by reviewing the distribution of Age-Specific Fertility Rates for Wisconsin's 72 counties and limiting outliers. For all four age groups, when the maximum historical Age-Specific Fertility Rate exceeded the stated cap, the maximum historical Age-Specific Fertility Rate was substituted for the stated cap.

Fertility: Notes about Calculating Birth Numbers

The population projection interval (4/1/2020-3/31/2030) was split into two 5-year intervals (4/1/2020-3/31/2025 and 4/1/2025-3/31/2030). Using the 2020 base population and the 2030 projected population, the female population was interpolated for the midpoint of each 5-year interval.

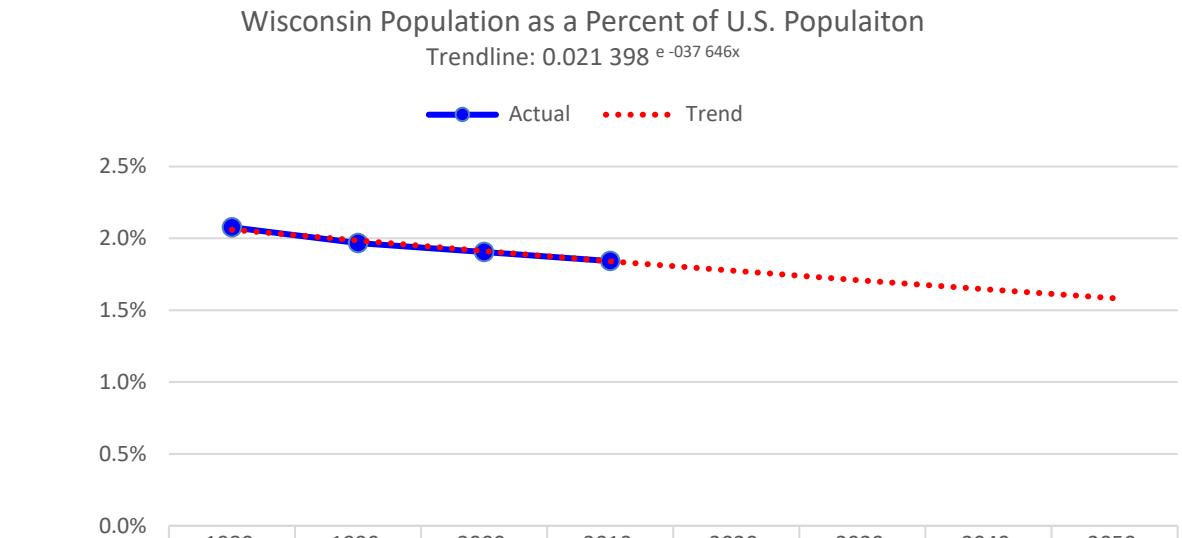
When Age-Specific Fertility Rates were multiplied by the number of females in the corresponding age group, this produces total birth figures. The Hamilton-Perry method requires distinct figures for female births and male births, so the 2011-2020 sex ratios were used (in aggregate 0.511791 male and 0.488209 female).

After calculating the number of births in the first 5-year interval (4/1/2020-3/31/2025), this birth number is multiplied by a survival rate to determine how many will be aged 5-9 in 4/1/2030. For people aged 0-4 and people aged 5-9, the Hamilton-Perry method does not account for migration.

State Control Total

When using the Hamilton-Perry method, it is generally recommended to use a different method to develop a state population projection and to use that state population projection as a control total, forcing county population projections to add to the state total. When calculating Wisconsin's population as a percent of U.S. population,[10] a reasonably stable trend emerges. (See graph immediately below.)

[10] For source data (Wisconsin population and U.S. population), see the National Historic Geographic Information System web site, https://www.nhgis.org/. (This is a resource provided by the Integrated Public Use Microdata System, IPUMS.)



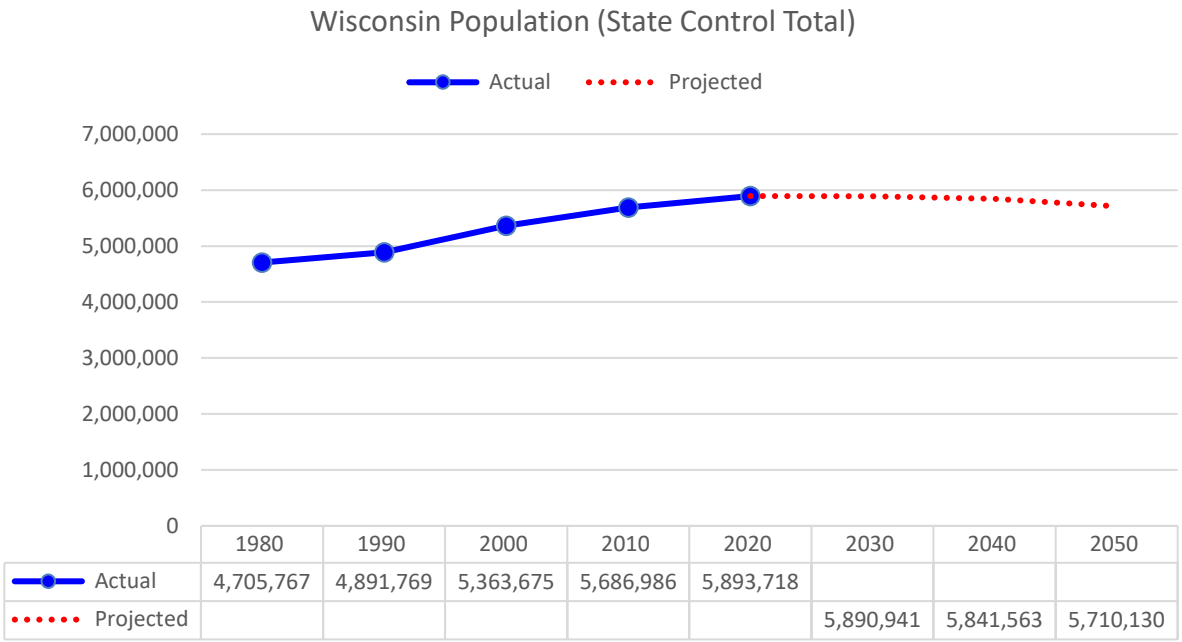
	1980	1990	2000	2010	2020	2030	2040	2050
Actual	2.077 181%	1.966 858%	1.905 920%	1.841 965%				
Trend	2.060 726%	1.984 590%	1.911 266%	1.840 651%	1.772 646%	1.707 153%	1.644 080%	1.583 337%

In 2023, the U.S. Census Bureau published national population projections.[11] The graph immediately above shows the trend of Wisconsin population as a percent of U.S. population. These percentages are applied to those U.S. Census Bureau population projections to produce Wisconsin statewide population totals. These Wisconsin totals are used as control totals. County projections were proportionally decreased (0.986580 in 2030) and proportionally increased (1.001520 in 2040 and 1.00385 in 2050) to match these state control total figures.

Rounding to Nearest 5: County Age-Sex Cohorts Rounded and Summed

This document describes math that was done with unrounded fractions. For county projections, the finest level of detail was a single age-sex cohort (for example, Brown County females aged 30-34). Once projections results were calculated for each cohort, the results were rounded to the nearest 5. Each county's rounded cohort results were summed to the county's total population. Next, all 72 Wisconsin counties' cohorts (for example all 72 figures for females aged 30-34) were summed to the rounded Wisconsin total. This process will change the state totals very slightly from those described in the State Control Total section.

[11] See generally https://www.census.gov/data/tables/2023/demo/popproj/2023-summary-tables.html. Specifically see https://www2.census.gov/programs-surveys/popproj/tables/2023/2023-summary-tables/np2023-t1.xlsx.



	1980	1990	2000	2010	2020	2030	2040	2050
Actual	4,705,767	4,891,769	5,363,675	5,686,986	5,893,718			
Projected						5,890,941	5,841,563	5,710,130

Minor Civil Division Population Projections

The Hamilton-Perry method used for county projections is less well suited to Minor Civil Division population projections for two reasons.[12] First, Wisconsin's Department of Health Services does not offer birth or fertility data at the Minor Civil Division level. Second, the 2020 Census employed a Disclosure Avoidance technique called Differential Privacy that allows reasonably high confidence in age distributions for high-population areas but allows lower confidence in very low-population areas (like some of Wisconsin's Minor Civil Divisions).

Minor Civil Division population projections are largely based on two numeric annualized growth figures: the MCD's numeric annualized growth from the 4/1/2010 count to the 4/1/2020 count, and the MCD's numeric annualized growth from the 4/1/2020 count to the 1/1/2023 estimate. These two numeric figures are averaged, and the resulting annualized growth figure is carried through the projection period (4/1/2020-4/1/2030). The provisional 2030 Minor Civil Division projections in each county are totaled to the county level. These totals are compared to the county totals from the county population projections, and the Minor Civil Division population projections for each county are proportionally increased or decreased to match the county projections.

[12] A Minor Civil Division is the portion of a municipality within one county. If the municipality is entirely within a single county, the Minor Civil Division is the same thing as the municipality. The Village of Pulaski is in Brown, Oconto, and Shawano counties, so it consists of three different Minor Civil Divisions. Sometimes it is not possible to sum municipal population to county population. Minor Civil Division populations can always be summed to county population.

The Hamilton-Perry method used for county population projections is more rigorous and reliable because it incorporates age distribution data and projects Age-Specific Fertility Rates, while the pre-raking Minor Civil Division projections do neither. Also, the U.S. Census Bureau's decision to implement Differential Privacy in its 2020 counts will have less impact on total population for higher-population areas like (most) counties and more impact on lower-population areas like (many) Minor Civil Divisions.

Adjustments were required in some cases. Numeric annualized growth figures were adjusted when distorted by anomalous events like large one-time population change due to a group quarters facility opening/closing/increasing/decreasing, a large partial annexation, or complete absorption of a neighboring MCD.

All Minor Civil Division calculations used unrounded figures. When figures were rounded for publication, the first iteration of rounded figures sometimes failed to match the county totals previously established (absolute differences were never more than four residents). If a gap existed, it was closed by adding to or subtracting from the county's most-populous Minor Civil Division.